where μ_0 is the permittivity of free space. In the constant velocity approximation this magnetic energy is supplied by the Poynting flux:

$$(1/\mu_0)(\boldsymbol{E} \times \boldsymbol{B}) \cdot \boldsymbol{A} = (1/\mu_0)B^2\boldsymbol{v} \cdot \boldsymbol{A} \tag{9}$$

where $\boldsymbol{A}$ is the area across which the plasma flux is supplied.

Conclusions

The rigorous analytical derivation shows that $1/r$ scaling is consistent with magnetic field lines "frozen" in a spherically expanding plasma and not just a result of a numerical simulation. The magnetic field energy is imparted at the source and does not rely on the properties of an interacting solar wind for additional energy. However, the derivation is idealized and still leaves open critical questions concerning the application of M2P2 to real spacecraft propulsion.

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Algebraic Correlation for High-Speed Transition Prediction on Sphere Cones

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Introduction

THE prediction of the occurrence of boundary-layer transition remains as one of the most challenging problems in basic physics. Transition occurs when disturbances present in a laminar flow are amplified and grow to the critical amplitude necessary for breakdown to a turbulent flow. The complexity of this problem is compounded by some unusual reversals in transition behavior for hypersonic flow. A discussion of the many paradoxes associated with high-speed transition may be found in Ref. 1. At present, there are no general empirically derived correlations for predicting hypersonic transition. Limited success in correlating transition data has been achieved by restricting the empirical database and thereby limiting the generality of the correlations. However, there are points in some of these correlations where the spread in uncertainty can vary as much as an order of magnitude.[2]

Future opportunities for acquiring high-speed boundary-layer transition data through flight testing will be rare due to the exceedingly high costs involved. Thus, the current challenge is to exploit the existing experimental database and to develop a method that reliably predicts the onset and movement of boundary-layer transition. The purpose of this Note is to describe a simple algebraic correlation for predicting laminar-to-turbulent flow transition on the frustum of smooth, nonablating sphere cones in free flight. This correlation is based on an analysis of a large set of well-documented high-speed ground- and flight-test data using a transition Reynolds number with a length scale equal to $(\theta^3/S)^{1/2}$, where θ is the momentum thickness and S is the body-surface streamlength. Because most reentry configurations employ an ablative thermal protection system, the applicability of the present correlation to bodies with ablating surfaces is examined.

Analysis and Data Selection

Inconsistencies in the data analysis can lead to scatter in the correlation process. For example, different investigators will employ different methods to predict edge conditions and, hence, will compute different local edge Reynolds numbers. Thus, transition location is not a well-defined quantity when reported in terms of a transition Reynolds number. For this reason, only results from experimental investigations that report actual transition locations are utilized. The most accurate method for determining transition onset location is believed to be onboard measurement of the sudden rise in surface heat transfer rates. Consideration for data selection required this method of transition detection for reentry flight tests. For ballistics-range ground tests, however, transition locations are typically estimated from shadowgraphs during low-Mach-number runs or from drag measurements during high-Mach-number runs.

Ballistics-range data are invaluable because they are free of the acoustic disturbances that contaminate conventional shock- and wind-tunnel measurements. However, results from shadowgraphs generally yield transition locations roughly halfway between onset and end.[3] Transition onset locations can be estimated from optically determined shadowgraph results using information given in Refs. 3 and 4. A comparison of transition detection methods given in Ref. 3 shows that peak temperature recovery factor locations roughly coincide with average transition streamlengths determined from shadowgraphs, that is, $S_{\text{peak}} \approx S_{\text{optical}}$. In addition, quiet-tunnel data from Ref. 4 suggest that the value of $S_{\text{onset}}/S_{\text{peak}}$ approaches 0.8 for high edge unit Reynolds numbers. This implies that $S_{\text{onset}}/S_{\text{optical}} \approx 0.8$. Although no analogous correction exists for drag-inferred transition data, they are included here as reported in the literature to increase the high freestream Mach number statistics associated with the database.

In addition, most reentry applications employ a nosetip made from an ablating material to withstand the high temperatures generated near the stagnation point during flight. An additional complication can be avoided by restricting the data selection to geometries with one type of nosetip material. This restriction is not a severe one because the vast majority of the available flights employed some type of graphitic material. Thus, only those reentry configurations with a graphitic nosetip are included in the present empirical database.

Other requirements, which substantially reduce the size of the available database, include 1) a sphere–cone configuration having a nonablating frustum material; 2) a high-speed free-flight flow environment, where the freestream Mach number is greater than 3.5; 3) a small total angle of attack at transition, namely, less than 1.25 deg for ballistics-range data and less than one-tenth of the half-cone angle for reentry data; and 4) a reentry or descent-phase laminar-to-turbulent flow transition. The final empirical database consists of 101 points, which were selected from the ballistics-range results of Potter[5] (5 points), Reda[6] (26 points), and Sheetz[7] (21 points) and the flight-test data of Berkowitz et al.[2] (32 points), Johnson et al.[8] (8 points), and Krasnican and Rabb[9] (9 points). This database covers a wide parameter space in terms of nose radius, half-cone angle, and freestream conditions. With respect to geometric parameters, the value of nose radius varies from 0.001 to 4 in. (0.00254 to 10.16 cm), whereas the value of half-cone angle varies from 5 to 22 deg. As for freestream conditions, the Mach number varies from 3.5 to 23.1, whereas the unit Reynolds number varies from 0.7×10^6/ft to 76.7×10^6/ft (2.3×10^6/m to 251.6×10^6/m).

The goal here is to determine those parameters that accurately correlate frustum transition. This is not an easy task because there have been many previous attempts to correlate transition data. A good example of the difficulty involved may be found in Ref. 2, where

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over 40 different correlation techniques were attempted, and only marginal success was achieved. Through a trial-and-error process, all of the points in the database were found to correlate remarkably well using the following transition Reynolds number as the dependent parameter:

$$Re_T = (\rho_e u_e/\mu_e)\left(\theta_T^3 \big/ S_T\right)^{\frac{1}{2}}$$

where ρ is the density, u is the velocity, μ is the viscosity, and the subscripts e and T refer to conditions at the boundary-layer edge and at transition onset. The corresponding independent parameter was the following nose Reynolds number:

$$Re_{N,w} = Re_{N,\infty}(\mu_\infty/\mu_w) = (\rho_\infty u_\infty/\mu_\infty)\left(R_N^3 \big/ S\right)^{\frac{1}{2}}(\mu_\infty/\mu_w)$$

where R_N is the nose radius and the subscripts ∞ and w refer to conditions in the freestream and at the wall, respectively.

Results

It is imperative to have a consistent method for the calculation of local boundary-layer edge properties and of integral parameters such as momentum thickness. Values of the Reynolds number Re_T for all points in the empirical database were generated using the 3DV computer code. A complete discussion of the accuracy and limitations of 3DV may be found elsewhere.[10] In correlating the empirical database, the following simplifications were made. The ballistics-range datasets[5–7] and the flight-test database compiled by Berkowitz et al.[2] do not provide wall temperature distributions. For these datasets, boundary-layer calculations were performed, which assumed a constant wall temperature equal to the initial temperature for the ballistics-range data points or to the transition onset values listed in Ref. 2 for the flight-test data points. In addition, there is no information on nosetip–frustum interface locations for the Berkowitz et al.[2] dataset to determine boundary-layer mass-addition effects due to nosetip ablation. To maintain a reasonable modeling approach, all reentry flight-test calculations were performed using the reported ablated nose radius and assuming any mass-addition effects on the Reynolds number Re_T to be negligible.

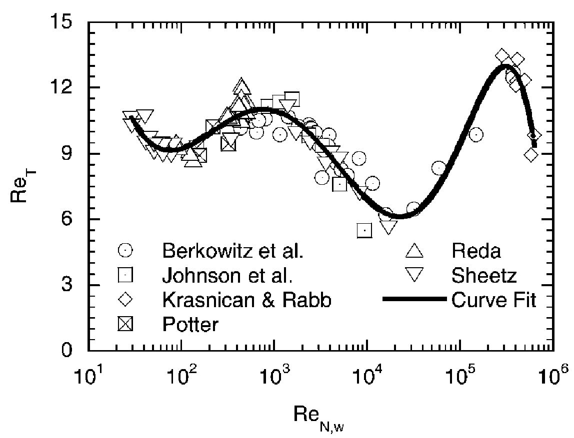

Fig. 1 Transition Reynolds number Re_T vs nose Reynolds number $Re_{N,w}$ for the entire empirical database of 101 points.

The results of the correlation analysis are summarized in Fig. 1. It is seen that good results are achieved using the correlation parameters described earlier. All of the ballistics-range data for nominally sharp bodies, that is, for $Re_{N,w} < 1000$, are consistent with each other and merge smoothly into both the ballistics-range and flight-test data points with larger effective nose radii. These results not only show that there is an identifiable influence of nose bluntness on transition, but also that ballistics-range transition data are directly relatable to flight-test transition data. On the other hand, these results do not offer an explanation or a better understanding of the mechanisms that cause transition. They, however, do provide a way of predicting the effect of bluntness on transition. The least-squares curve fit

$$Re_T = \sum_{i=0}^{7} a_i[\log(Re_{N,w})]^i$$

represents the final results with a correlation coefficient of 0.925. The values of the individual coefficients are $a_0 = 57.312$, $a_1 = -53.577$, $a_2 = 7.7869$, $a_3 = 9.1791$, $a_4 = -3.1201$, $a_5 = -0.057287$, $a_6 = 0.11973$, and $a_7 = -0.011056$.

Applicability to Ablating Bodies

Algebraic correlations are often unreliable because they cannot account for all of the numerous parameters that affect transition. Accurate results most always will be obtained when the geometry and flow conditions are similar to those on which the correlation is based. The utility of the present correlation appears to be restricted because many hypersonic applications employ an ablative system to protect internal components. To assess the predictive capability of the present correlation with surface mass-addition effects, comparisons will be made against experimental results obtained on ablating 30-deg half-angle sphere cones in a ballistics-range environment.[11,12] The models used in these tests were recovered intact, making it possible to determine the amount of mass ablated and any surface patterns consistent with the spread of turbulence.

In this Note, eight runs that experienced mass loss amounts consistent with laminar heat transfer rates are considered. Boundary-layer transition predictions were made using the present correlation and three other empirical correlations to demonstrate the wide variation in results that is commonly encountered. The other empirical correlations are 1) the momentum-thickness Reynolds number over edge Mach number correlation,[13] $Re_\theta/M_e = 100$; 2) the low recession nosetip (LORN) correlation,[14] $Re_\theta = 275\exp(0.134M_e)$; and 3) the General Electric low mass-addition (GELMA) correlation,[15] $Re_S = 50 \times 10^5$ for $M_e < 1.4$ or $Re_S = 2.42 \times 10^5 M_e^{1.915}$ for $M_e > 1.4$. All of the predictions assumed an initial nosetip radius of 0.001 in. (0.00254 cm), a freestream velocity equal to the launch velocity, and a constant wall temperature of 540°R (300 K).

As can be seen by the results summarized in Table 1, none of the empirical correlations predicts a fully laminar flow for all eight runs. The results from the present correlation are seen to provide

Table 1 Comparison of predicted normalized transition location using various empirical correlations for ablating sphere cones in a ballistics-range environment

				Normalized transition location, x_T/L			
Run	Model	p_∞, atm	v_∞, m/s	Re_θ/M_e correlation	LORN correlation	GELMA correlation	Present correlation
1	Lexan	1	1949	0.2203	0.4394	0.7329	>1.0
2	Lexan	1	3200	0.2161	0.3509	0.7084	0.9337
3	Lexan	2	3660	0.1071	0.1648	0.3564	0.5713
4	Delrin	1	3000	0.2157	0.3595	0.7097	0.9467
5	Delrin	1	3057	0.2152	0.3560	0.7075	0.9438
6	Delrin	1	3639	0.2186	0.3402	0.7132	0.9258
7	Delrin	1	5686	0.2402	0.3235	0.7692	0.9011
8	Delrin	1	5771	0.2406	0.3225	0.7702	0.8912

a good overall estimate of the boundary-layer state. For seven of the eight runs, the present correlation yields a laminar-flow state for at least nine-tenths of the body length L. For these same runs, the GELMA correlation predicts a laminar-flow condition for at least seven-tenths of the body length, whereas the Re_θ/M_e and LORN correlations give much shorter laminar run lengths. All of the correlations, however, predict an extensive region of turbulent flow over the body for run 3. A later examination of the ablation patterns for this run[12] revealed surface crosshatching on the model. This suggests that turbulence existed at some time during the run but not at a level significant enough to promote higher than fully laminar mass loss.

Conclusions

An algebraic correlation that predicts the onset and movement of frustum boundary-layer transition on smooth, nonablating sphere cones in high-speed flow has been presented. This correlation was developed from an analysis of a large set of well-documented free-flight test data. The key to the analysis was the use of a transition Reynolds number with a length scale equal to $(\theta^3/S)^{1/2}$ as the correlating parameter. The present correlation displays an identifiable influence of bluntness on transition behavior and is superior to other correlations for predicting the boundary-layer state on bodies with small amounts of mass loss due to ablation.

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